Note: Exam 2 is Thursday!!!

Covers 6.4, 6.5, 7.1-7.5, 7.7, 7.8 The exam will roughly look like this: 3 pages: 6 integrals (*ALL* types) 2 pages: 6.4, 6.5, 7.7 and/or 7.8 (8.1 Arc Length is NOT on our midterm this quarter)

#### A Brief Exam 2 Review

Integration! Work. Average Value. Trapezoid, Simpson Rules. Improper Integrals.

#### 1. The 4 special methods

By parts:  $xe^{3x}, x^2 \cos(5x),$  $\frac{\ln(x-1)}{x^2}, x \tan^{-1}(x).$ 

Trig:

$$sin^{3}(x) cos(x), cos^{4}(x),$$
  

$$sec^{3}(2x) tan^{3}(2x)$$

Trig sub:

$$\frac{1}{x\sqrt{x^2-9}}, \frac{1}{(4-x^2)^{3/2}}, \frac{1}{\sqrt{x^2+6x+10}}.$$

Part Frac:

$$\frac{x+2}{(x-1)(x-3)'} \frac{4x}{(x-1)^2(x-3)'} \\ \frac{5}{x(x^2+4)'} \frac{x^2}{x+7} \frac{x-3}{x^2+8x+20}$$

2. Substitution and Simplifying

Try  $u = \sqrt{x}$ , u = inside, u = e<sup>x</sup>, u = trig.

Completing the square.

Trig facts.

Square identities, half-angle. Triangle trick.

$$1.\int \frac{1-x}{\sqrt{1-x^2}} dx$$

$$2.\int \frac{x^2 - x + 8}{x^3 + 4x} dx$$

$$3.\int 2x\ln(x+5)\,dx$$

$$4.\int\cos^3(x)\,dx$$

$$5. \int_0^2 \frac{1}{\sqrt{x^2 + 2x + 4}} \, dx$$

$$6. \int_{1}^{3} \frac{1}{x^2 + x^3} dx$$

$$7.\int \tan^2 x \sec^4(x) \, dx$$

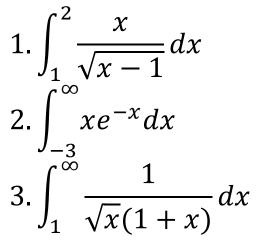
$$8.\int \frac{1}{\left(1+\sqrt{x}\right)^3} dx$$

$$9.\int \sin(x)\sqrt{\cos(x)}dx$$

## 3. Improper Integrals:

- a) Rewrite as a limit!!
- b) Integrate
- c) Take limit

## **Random Improper Integrals:**



# 4. Trapezoid/Simpson Rules

a) Set up integral, then compute widthand label tickmarks.b) Use formula.

## **Approximation Example:**

1. Use Simpson's Method with n = 4 subdivision to approximate the value of

$$\int_{0}^{4} \sqrt{1+4x^4} dx$$

**5. New Applications** 

a) Average value =  $\frac{1}{b-a} \int_{a}^{b} f(x) dx$ 

b) Work =  $\int_{a}^{b} (Force)(Dist)$ Step 1: Draw picture (start and end) Label clearly.

Draw a typical subdivision.

Step 2: Find pattern for Force and Dist.

Step 3: Integrate.

Type 1: "Changing force" Force changing as object is moved (leaky bucket, springs, given force). f(x) = "force formula at x" Force = f(x), Dist =  $\Delta x$ ; Work =  $\int_{a}^{b} f(x) dx$  Type 2: "Stack of books" (chain, pumping) Chain/Cable: Given k = density = force/length if x = 0 is labeled at the top. then for any subdivision that makes it to the top: Force = k  $\Delta x$ , Dist = x Work =  $\int_{a}^{b} k x dx$ 

Pumping:

Given k = density = force/volume if top is y = b, then Force = k(Area) $\Delta y$ , Dist = b - y; Work =  $\int_{a}^{b} k(Area)(b - y)dy$ 

You need to have an basic understanding of how we label and find pattern to be able to adapt to changes in these problems!!

#### **Applications from old tests:**

1. Find the average value of  $\cos^3(x)$  on the interval 0 to  $\pi/2$ .

3. A 1600 lb elevator is suspended by a 200 ft cable that weighs 10 lb/ft.How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft? 4. A rope is used to pull a bucket full of water up from a well that is 10 m deep. The rope has a total mass of 5 kg. The bucket has a mass of 11 kg. Find the total work done in lifting the bucket to the top (Recall: Accel. due to gravity is 9.8 m/s<sup>2</sup>) 5. A well is in the shape of a cylinder of radius 1 meter and depth 8 meters.
It is half full of water. Find the word to pump all the water to the top.
(Recall: Water weighs 9800 N/m<sup>3</sup>)

6. The portion of the graph  $y = x^2 / 9$ between x = 0 and x = 3 is rotated about the y-axis to form a container. The container is full of a liquid that has density 100 lbs/ft<sup>3</sup>. Find the work required to pump all the liquid to the top of the container.